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Cycle Analysis: The Moving Average

By Edward R. Dewey
Director, Foundation for the Study of Cycles

The moving average is a mathematical tool of great use to students of cycles. As there is confusion in the minds of some people in regard to the use of this tool, it seems wise to issue a bulletin on the subject.

Some sections of this bulletin are merely a restatement of what you can find in any good text book of statistics. In other sections, however, you will find material, some of which is not, as far as I know, available readily, if at all.

I. DEFINITIONS AND DESCRIPTION OF METHODS

(The Simple Arithmetic Moving Average)

Averages

Everybody knows that an **average** is a typical value which tends to sum up or describe a number of figures. There are at least five different kinds of averages commonly used by statisticians; but the one which ordinary folk think about when they hear the word **average** is the one computed by adding all the items together and dividing the total by the number of items. Thus, if we have four items, 10, 12, 11, and 13, the average of these items is 10 + 12 + 11 + 13 (46) divided by 4, or 11½. (Statisticians call an average computed this way the **arithmetic mean**, but you do not need to remember this term, because I shall not use it again.)

Time Series

An arrangement of numbers is called a **series**. When the numbers with which we deal represent events which occur one after another in time, the arrangement is called a **time series**. Thus, in the example above, if 10, 12, 11, and 13 represent the price of cotton for each of four consecutive years, or represent the number of accidents on each of four consecutive days, you would call the numbers by this name—a time series.

You could still average the numbers and say, for example, that the average price for all four years was 11½ cents, or that during the period there was an average of 11½ accidents per day, as the case might be.

You could also say that the average price for the first three years was 11 cents, (10 + 12 + 11 (33) divided by 3) and that the average price for the last three years was 12 cents (12 + 11 + 13 (36) divided by 3).

Moving Averages

A moving average is merely a succession of averages secured from a series of numbers by dropping the first number (item) in each group averaged and including the next number in the series after the group, thus obtaining the next group to be averaged, and so on.

Thus, when you averaged the first three numbers of our time series (10, 12, and 11) and got 11, and then dropped the first number (10) and added the fourth number (13) and averaged again and got 12, you were constructing a moving average. Easy, wasn't it?

Because you were averaging three items at a time, you would call the result a 3-item or 3-term moving average. If the items represented yearly values you would call the result a 3-year moving average. If the items represented daily values, you would call the result a 3-day moving average.

Moving Totals

The moving total is the series of successive totals from which the moving average is computed.

For example: When, above, you added 10, 12, and 11 to get 33, and then added 12, 11, and 13 to get 36 (as a step in the task of getting 11 and 12, the two terms of the moving average), you were computing a moving total.

It was so easy that you did it without knowing it!

The moving total, like the moving average, should be posted in a table or plotted on a chart against the middle item of the group of items being totaled, as will be explained below.

Plotting or Posting Moving Averages

Each item or term of a moving average is always properly posted or plotted against the center of the group of items being averaged.

Many otherwise intelligent people fool themselves into thinking that if they plot or post an average against the last figure of the group of figures being averaged they somehow are getting later values. Of course this is nonsense.

If the values for 1933, 1934 and 1935 were 10, 12, and 11 respectively, the average for these three years is 11, whether we say 11 for the three years **beginning** in 1933, or 11 for the three years **centering** on 1934, or 11 for the three years **ending** in 1935. In **talking** about an average, we could choose any one of the three ways with equal propriety, as long as we made it clear which way we had chosen. But when averages are posted to a table, or plotted as a point on a chart, they **must** be posted or plotted against the middle of the group of figures being averaged, otherwise convention will be violated and, much more important, distortions are introduced into all further work. (The reasons for this will appear later.) Let me repeat, moving averages must **always** be posted or plotted against the central item of the items being averaged.

Two Examples

To make the process doubly clear, let us work out teo examples:

TABLE 1.
COMPUTATION OF 3-YEAR AND OF 7-YEAR MOVING AVERAGES

		COMPUTATION OF A 3-YEAR MOV. AVER.		COMPUTATION OF A 7-YEAR MOV. AVER.	
	A	B	C	D	E
YEAR	DATA	3-YEAR MOVING TOT. OF COL. A	3-YEAR MOVING AV. OF COL. A (COL. B÷3 OR COL. B x 1/3)	7-YEAR MOVING TOT. OF COL. A	7-YEAR MOVING AV. OF COL. A (COL. D÷7 OR COL. D x 1/7)
1933	10	-	-	-	-
1934	12	33	11	-	-
1935	11	36	12	-	-
1936	13	40	13 1/3	91	13
1937	16	43	14 1/3	98	14
1938	14	45	15	109	15 4/7
1939	15	46	15 1/3	127	18 1/7
1940	17	55	18 1/3	149	21 2/7
1941	23	69	23	173	24 5/7
1942	29	87	29	-	-
1943	35	104	34 2/3	-	-
1944	40	-	-	-	-

It is obvious that with a 3-year moving average there are no values to place against the first and the last items of the series. With a 7-year moving average there are no values to place against the three first and the three last items of the series. In constructing a moving average one always loses one or more terms at each end.

Formulae

The formula for a 3-year moving average is

$$MA_b = \frac{a + b + c}{3}$$

where a to c represent successively each three consecutive terms of the data and MA_b stands for the 3-year moving average to be posted against the central term, b.

The formula for a 7-year moving average would be

$$MA_d = \frac{a + b + c + d + e + f + g}{7}$$

where a to g represent successively each seven consecutive terms of the data and MA_d stands for the 7-year moving average to be posted against the central term, d.

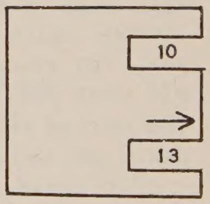
Mechanical Details of Computation

As has been explained, to get a 3-year moving average, one first computes a 3-year moving total, and divides each item of the moving total by 3.

To get the first figure of the moving total, add together the first three items of the data. To get the next figure of the moving total you subtract the first item of the data, and add the fourth. This process gives you the sum of items 2, 3, and 4. You proceed in this way successively.

In actual practice it is hard to pick out which items to add and which to subtract. You get mixed up.

To make the calculation foolproof, cut two slots out of a card or piece of paper so as to expose the first and fourth items in the series, but not the second and third. In our example in Table 1, it would look like this: (The lines **between** slots must always be one less than the number of terms in the moving average.)



Place this screen over the data so that the first item (in this instance 10) appears in the upper slot and the fourth item (in this instance 13) in the lower one.

Now, from 33, the sum of the first three figures, already posted in Col. B, subtract whatever appears in the upper slot (10) and add what you see in the lower slot. This gives you 36 which you enter in Col. B opposite the arrow. (The arrow is placed against the middle figure of the three items whose total is obtained by this method.)

```
 10 *
 12
 11
 33 S
 10 -
 13
 36 S
 12 -
 16
 40 S
 11 -
 14
 43 S
 13 -
 15
 45 S
 16 -
 17
 46 S
 14 -
 23
 55 S
 15 -
 29
 69 S
 17 -
 35
 87 S
 23 -
 40
104 *
=====
 29
 35
 40
104 *
=====
```

Now, slip your screen down a line so that 12 shows in the upper slot and 16 in the lower one.

From 36, subtract 12 and add 16 to get 40, the third item of your moving total.

Continue in this way until you have dropped 23 and added 40 to come up with the final item in the moving total, namely 104.

Now, add together the last three items of the data-- 29 + 35 + 40, to get 104 as a check on the accuracy of your work.

If you use an adding machine with direct subtraction, your tape will look like the figures shown to the left:

I find it better to run the entire tape before posting any values to Col. B. One reason is that it is quicker to do your posting all at once. Another reason is, if you should make an error you will not need to erase from Col. B all the figures from the error forward.

In doing long columns of figures, I also find it a good idea to check every 50 or 100 items by adding up the proper number of items of the original data to see if the total agrees with the subtotal on my tape.

Now that we have our moving totals (Col. B) we compute the moving average either by dividing each figure of the moving total by 3 or by multiplying it by the reciprocal of 3. This latter method is often easier. The reciprocal of a number is 1 divided by the number. In this case it is .333333.

Actually, .333333 x 33, the first figure in Col. B is 10.999989 which, of course, rounds to 11. To get the 11 in the machine directly, I always record the last digit of the reciprocal as one more than it really is. In this case we would therefore have .333334 x 33 or 11.000022. The error is twice as large, but the excess is dropped anyway and this method saves the need of rounding.

Alternate (Short-Cut) Method

You can compute a moving average directly without computing the moving total. When a calculating machine is available, this method is usually preferable. The method is a little hard to describe but very easy to compute. Proceed as follows:

Place the reciprocal of the number of items of the moving average in the machine. As we are computing a 3-item moving average, we put in the machine the reciprocal of 3 which is 1/3 or .333333 (only, as above, we call it .333334). We lock this figure into the machine for the whole operation.

We first multiply this reciprocal by the first item of our data, 10, and obtain 3.33334. **Without removing this product**, we then multiply the reciprocal by 12 (add it in 12 times) and obtain a total of 7.333348. **Without removing the product** we then multiply the reciprocal by 11 to obtain a grand total of 11.000022 or 11, which is the first figure of our moving average. (1/3 of the first item + 1/3 of the second item + 1/3 of the third item is the same as the sum of the first three items divided by 3.)

We then remove 1/3 of the first figure by subtracting the locked-in reciprocal 10 times to get 7.666682 and multiply (add the reciprocal in) 13 times to obtain 12.000024 or 12, the second figure of our moving average. This process is continued right down the column until 23 times the reciprocal has been removed and 40 times the reciprocal has been added in to obtain 34.666736 or 34 2/3 for the moving average value for 1943. This value is checked by adding together 29 times the reciprocal, 35 times the reciprocal, and 40 times the reciprocal, or adding 29, 35, and 40 and dividing by three.

If we were computing a 7-year moving average, we would, of course, use the reciprocal of 7, which is .142858 (the last figure has been raised by one). To get the first item (or term as it is more usually called) of our moving average, we add together the sum of this reciprocal times each of the first seven items of the data, and then add and subtract products of the reciprocal as above.

Moving Averages With an Even Number of Items

You may have noticed that so far we have talked exclusively about moving averages with an odd number of terms—3 or 7.

When we compute moving averages with an even number of terms such as 2 or 4, we run into a slight complication, due to the fact that the moving average must always be posted or plotted against the middle of the group of data being averaged, and the middle of an even number of items fall between two of the items.

We could post or plot a 4-year moving average **between** the years and this is sometimes done, as in the table on the following page:

TABLE 2.
COMPUTATION OF A 4-YEAR MOVING AVERAGE

Year	A Data	B 4-Year Mov. Tot. of Col. A	C 4-Year Mov. Aver. Of Col. A (Col. B÷4 or Col. B x .25)
1933	10		
1934	12		
		46	11½ (At position 1934½)
1935	11		
		52	13 (At position 1935½)
1936	13		
1937	16		

However, the results of this method of posting are very awkward to describe in words and preclude any comparison between the moving average and the original data. Therefore, in practice it is almost **universal** to compute a 2-item moving average of the even-term moving average in order to center the moving average exactly, thus:

TABLE 3.
COMPUTATION OF A 2-YEAR MOVING AVERAGE OF A 4-YEAR MOVING AVERAGE

Year	A Data	B 4-Year Moving Tot. of Col. A	C 4-Year Moving Av. of Col. A (Col. B÷4 or Col. B x .25)	D 2-Year Moving Tot. of Col. C	E 2-Year Moving Av. of Col. D (Col. D÷2 or Col. D x .5)
1933	10				
1934	12				
		46	11½		
1935	11			24½	12¼
		52	13		
1936	13			26½	13¼
		54	13½		
1937	16				
1938	14				

You note that the first item of the 2-year moving average of the 4-year moving average is centered exactly against 1935; the second item is centered exactly against 1936.

In practice one would have computed a 4-year moving total, then a 2-year moving total of the 4-year moving total, and divided these values by 8, thus:

TABLE 4.
COMPUTATION OF 2-YEAR MOVING AVERAGE OF 4-YEAR MOVING AVERAGE—PREFERRED METHOD

Year	A Data	B 4-Year Moving Tot. of Col. A	C 2-Year Moving Tot. of Col. B	D 2-Year Mov. Aver. of 4-Year Mov. Av. of Col. A (Col. C ÷ 8)
1933	10			
1934	12			
		46		
1935	11		98	12¼
		52		
1936	13		106	13¼
		54		
1937	16			
1938	14			

Or one would have used the short-cut method and computed a 4-year moving average directly, as explained above, without bothering with the 4-year moving total. One could also have computed the 2-year moving average of the 4-year moving average directly by the same means:

TABLE 5.
COMPUTATION OF 2-YEAR MOVING AVERAGE OF 4-YEAR MOVING AVERAGE—SHORT-CUT METHOD

Year	A Data	B 4-Year Mov. Average of Col. A, Computed Directly	C 2-Year Mov. Aver. of Col. B, i.e. a 4-Year Mov. Aver. of Col. A, Centered
1933	10		
1934	12		
		11½	
1935	11		12¼
		13	
1936	13		13¼
		13½	
1937	16		
1938	14		

Also, in actual practice, to save space, one posts the 4-year moving average in either the second or third position but marks it clearly to indicate that it is not truly centered as it should be, thus:

TABLE 6.
COMPUTATION OF THE 2-YEAR MOVING AVERAGE OF A 4-YEAR MOV. AVERAGE--POSTED AS IT IS DONE IN ACTUAL PRACTICE

	A	B	C
YEAR	DATA	4-YEAR MOV. AV. POSTED TO THE SECOND POSITION (CENTERED MINUS ½ YEAR)	2-YEAR MOV. AVER. OF 4-YEAR MO. AV. I.E. A CENTERED 4-YEAR MOV. AVER. OF COL. A
1933	10		
1934	12	46	
1935	11	52	12¼
1936	13	54	13¼
1937	16		
1938	14		

In any event, the final column is called a 2-year moving average of a 4-year moving average or, more usually, a centered 4-year moving average.

Formulae

For those who like to have relationships expressed in formula form, it may be stated that the formula for a centered 4-year moving average is:

$$MA_c = \frac{a + 2b + 2c + 2d + e}{8}$$

or more simply:

$$MA_c = \frac{\frac{1}{2}a + b + c + d + \frac{1}{2}e}{4}$$

where a to e represent successively each five consecutive items of the data and MA_c stands for the 4-year moving average to be plotted against c, the center term or item.

II. THE USE OF THE MOVING AVERAGE

This second section of the report will tell you how to use the moving average in statistical procedure, with particular emphasis upon its use in cycle analysis.

The moving average is used (a) to smooth time series, (b) to approximate the trend of time series, and (c), in cycle analysis, to help us (i) to separate cycles and (ii) to obtain a more exact estimate of the characteristics of each of the various cycles that may be present.

A. The Use of the Moving Average to Smooth Time Series

The chief use of moving averages in ordinary statistical procedure is for the smoothing of time series. As this use of the moving average as such does not particularly concern the cycle analyst, it will be touched upon here only very briefly.

A smooth curve is one which does not change its slope in a sudden or erratic manner. The student interested in smoothing time series is referred to Frederick R. Macaulay's classic, **The Smoothing of Time Series**, published by the National Bureau of Economic Research (New York) in 1931. This book is now out of print but you can occasionally pick up a copy ($5 to $10) in second-hand book stores and of course you can always consult it at any good library.

The Effect of Moving Averages Upon Random Fluctuations

It should be obvious that the effect of moving averages upon random fluctuations is to average out the irregularities. It should be equally obvious that the more items that are combined into the moving average, the smoother will be your result and the closer it will approximate the average value of the successive numbers. One example should be enough to make this perfectly clear.

In Col. A of Table 7 are shown 30 digits taken at random from the New York City telephone book. For demonstration, these digits have been smoothed by a 3-item moving average, a 7-item moving average, and a centered 10-item moving average. See Fig. 1 on page 307.

It is obvious by inspection that as we increase the number of items of the moving average, the closer all terms of the moving average approach the average value of all the digits, which is 5.07.

TABLE 7.
VARIOUS MOV. AVERAGES OF A SERIES OF RANDOM NUMBERS

	A	B	C	D
ITEM	RANDOM NUMBERS (TAKEN FROM THE TELEPHONE BOOK)	3-ITEM MOV. AVER. OF COL. A	7-ITEM MOV. AVER. OF COL. A	CENTERED 16-ITEM MOV. AVER. OF COL. A
1	6	-	-	-
2	6	6.67	-	-
3	8	6.67	-	-
4	6	4.67	5.00	-
5	0	2.33	5.28	-
6	1	3.00	5.28	-
7	8	5.67	4.43	-
8	8	7.33	3.71	-
9	6	5.33	5.00	5.19
10	2	3.00	5.57	4.81
11	1	4.00	5.14	4.66
12	9	5.00	5.00	4.66
13	5	6.33	5.28	4.84
14	5	5.67	5.00	5.09
15	7	6.67	4.86	5.09
16	8	5.00	4.86	5.06
17	0	2.67	4.86	5.12
18	0	3.00	5.14	5.12
19	9	4.67	4.43	5.03
20	5	7.00	4.28	5.00
21	7	4.67	5.43	5.06
22	2	5.33	6.57	5.09
23	7	5.67	5.28	-
24	8	7.67	4.57	-
25	8	5.33	4.86	-
26	0	2.67	5.57	-
27	0	3.00	5.14	-
28	9	5.33	-	-
29	7	6.67	-	-
30	4	-	-	-

(SEE FIG. 1 ON PAGE 307)

Of course the process of smoothing also has the effect of minimizing cyclic fluctuations that may be present in the data as well as of smoothing out random fluctuations. It would not seem necessary to illustrate this fact at this point.

Weighted Moving Averages

In connection with smoothing a time series, one often gets better (i.e. smoother) results by the use of severak successive smoothings. For example, if one took a 2-year moving average of a 6-year moving average of a 9-year moving average of a time series, one would obtain a much smoother curve than could be obtained by any of these moving averages taken separately.

The compound effect of such a series of consecutive moving averages could be expressed by the following formula:

$$MA_h = \frac{a+3b+5c+7d+9e+11f+12g+12h+12i+11j+9k+7l+5m+3n+o}{108}$$

Such a moving average is called a weighted moving average because for each item of the moving average each of the terms is used a different number of times and therefore with different weights.

In the formula described, for any one term of the moving average each of the items g, h, and i have 12 times the effect or weight in the composite as do items a or o, which are used but once.

Macaulay reports upon many formulae which have been developed by various investigators in order to achieve particular purposes. For example, "....take a 3-months moving total of a 5-months moving total of an 8-months moving total of a 12-months moving total of the data. To the results apply the following extremely simple set of weights: + 2, -3, 0, 0, 0, 0, 0, + 3, 0, 0, 0, 0, 0, -3, + 2. Divide the final results by 1440."

A weighted moving average of the sort described above, **with negative weights** near the ends, if properly designed, will overcome the tendency of the ordinary moving average to stay too low at cycle tops and too high at cycle bottoms.

Some of these formulae become rather complicated For example:

> " Take successively a 3-months moving total of the data, a 5-months moving total, another 5, an 8, and a 12-months moving total. To the results apply the following set of weights: + 1,331,771, - 1,949,056, 0, 0, 0, 0, 0, + 2,175,370, 0, 0, 0, 0, 0, 0, - 1,949,056, + 1,331,771. Divide each of the final results by 6,773,760,000. "

In view of the fact that Macaulay has covered the subject so admirably and the further fact that we here are not interested in smoothing as such but only in the smoothing that may result as we use the moving average in the detection and isolation of cycles, it seems unnecessary to give further attention at this time to this aspect of the subject.

B. The Use of Moving Averages in Trend Determination

The moving average is often used to give an approximate measure of the trend of a time series.

Trend may be defined as the tendency of data in a series to increase or decrease over a long period of time. How long is " long " depends upon circumstances.

The table below shows, by means of controlled data, three trend lines and, in connection with each, its 5-year moving average.

The trend shown in Col. A increases by a **constant amount**. The trend shown in Col. C increases by **amounts that get progressively greater** as time goes on. The trend shown in Col. E increases by **amounts that get progressively less** as time goes on. Cols. B, D, and F show a 5-year moving average for each of these three trend lines. The trends, with their moving averages superimposed by means of broken lines, are shown in Fig. 2 on page 307.

TABLE 8.
5-YEAR MOVING AVERAGES OF CONTROLLED DATA SHOWING EFFECT UPON THREE DIFFERENT TYPES OF TREND

	A	B	C	D	E	F
YEAR	A TREND WHICH INCREASES BY A CONSTANT AMOUNT	5-YEAR MOVING AVERAGE OF COL. A	A TREND THAT INCREASES BY AMOUNTS THAT GET GREATER	5-YEAR MOVING AVERAGE OF COL. C	A TREND THAT INCREASES BY AMOUNTS THAT GET SMALLER	5-YEAR MOVING AVERAGE OF COL. E
1ST	40	-	0	-	0	-
2ND	80	-	5	-	80	-
3RD	120	120	15	20	155	150
4TH	160	160	30	35	225	220
5TH	200	200	50	55	290	285
6TH	240	240	75	80	350	345
7TH	280	280	105	110	405	400
8TH	320	320	140	145	455	450
9TH	360	360	180	185	500	495
10TH	400	400	225	230	540	535
11TH	440	440	275	280	575	570
12TH	480	480	330	335	605	600
13TH	520	520	390	395	630	625
14TH	560	560	455	460	650	645
15TH	600	600	525	530	665	660
16TH	640	-	600	-	675	-
17TH	680	-	680	-	680	-

(SEE FIG. 2 ON PAGE 307)

It will be noted by comparing the moving averages with the trend that where, as in Col. A, the trend increases by a constant amount the moving average coincides with it. Where, as in Col. C, the trend increases by amounts that get greater as we go from year to year, the moving average lies above the trend. Where, as in Col. E, the trend increases by amounts that get less from year to year the moving average lies below the trend.

The Geometric Moving Average and Its Use

For growth curves that increase by an increasing amount, such as the curve set forth in Col. C above, we can usually get a better fit by computing the **geometric moving average**.

In fact when the growth increases by increasing amounts such that the **rate** of growth is constant, the geometric moving average will give a perfect fit.

The geometric moving average is merely the **nth root** of the terms **multiplied** together instead of the **nth** of the terms **added** together. For example, for a 5-year **geometric** moving average, instead of successively **adding** together each five consecutive terms and **dividing** by five, you successively **multiply** together each five consecutive terms and take the fifth **root** of the product. The formulae for a 5-year arithmetic moving average and a 5-year geometric moving average are as follows:

The arithmetic moving average:

$$MA_c = \frac{a + b + c + d + e}{5}$$

The geometric moving average:

$$GMA_c = \sqrt[5]{a \times b \times c \times d \times e}$$

Of course, in practice, to get a geometric moving average, one merely looks up the loga-

rithms of the data in a table of logarithms, records them as in Col. B in Table 9, which follows below, computes the arithmetic moving average of the logs, and reconverts by looking up the antilogs, all as demonstrated in the table.

TABLE 9.
COMPUTATION OF A 5-YEAR GEOMETRIC MOVING AVERAGE CONTROLLED DATA

YEAR	A DATA (TREND, WITH CONSTANT 6% RATE OF GROWTH)	B LOGS OF COL. A	C 5-YEAR ARITHMETIC MOV. AVER. OF THE LOGS	D ANTILOGS OF COL. C I.E. 5-YEAR GEOMETRIC MOV. AVER. OF COL. A
1ST	100.00	2.0000	-	-
2ND	106.00	2.0253	-	-
3RD	112.36	2.0506	2.0506	112.36
4TH	119.10	2.0759	2.0759	119.10
5TH	126.25	2.1012	2.1012	126.25
6TH	133.82	2.1265	2.1265	133.82
7TH	141.85	2.1518	2.1518	141.85
8TH	150.36	2.1771	2.1771	150.36
9TH	159.38	2.2024	2.2024	159.38
10TH	168.95	2.2227	2.2227	168.95
11TH	179.08	2.2530	2.2530	179.08
12TH	189.83	2.2784	2.2784	189.83
13TH	201.22	2.3037	2.3037	201.22
14TH	213.29	2.3290	2.3290	213.29
15TH	226.01	2.3543	-	-
16TH	239.66	2.3796	-	-

As the 5-year geometric moving average is seen by inspection to be the same as the data, there seems to be no need to chart the result.

When the **rate** at which the curve increases is **decreasing**, the geometric moving average will lie below the curve. When the **rate** at which the curve is **increasing**, the geometric moving average lies above the curve. When the **rate** of growth is constant, as in the example above, the geometric moving average lies on the curve. I find the geometric moving average very useful, and use it a great deal.

Even though very few curves grow at an absolutely constant rate of growth, it is true that many growth curves tend to increase this way and are concave upward when plotted on arithmetic paper; in other words, they grow from year to year in an absolute amount which increases with each successive term. This is one reason why, in most cycle analyses it is usually desirable to deal with the logarithms of the data instead of with the data themselves.

C. The Use of the Moving Average in Cycle Analysis

How can a knowledge of moving averages be used to assist you in cycle analysis—that is, (i) to help you detect and separate cycles that may be present in the data you are studying, and (ii) to help you to obtain a more exact knowledge of their characteristics than would otherwise be possible?

Every time you compute a moving average of a time series you affect cycles of every length that may be present in that series. But, you influence cycles of different length in very different ways. And this fact in turn has an effect upon the comparison that you may make between two different moving averages or between the original data and the moving average.

Therefore, where there are several cycles present concurrently in a time series, by a suitable selection of moving averages, you can minimize or even eliminate some of these cycles and leave others virtually unchanged or, if you wish, magnified.

To see how to make these manipulations, you must first examine the effect of moving averages of different lengths upon a perfectly regular cycle that we can use for purposes of demonstration.

This brings up the question of the shape of the cycle that we should use. However, before we begin to talk about cycles and wave shapes, we will need to have in mind a few more definitions of terms. With these out of the way we can return to a discussion of the proper shape of wave to use for our demonstration, without the need of interrupting the discussion to define terms as we go along. From that point we can go on to a discussion of the effect of moving averages upon the wave shape we have chosen.

Definitions of Certain Terms Used in Cycle Analysis

Cycle, coming from a Greek word meaning circle, implies coming around to the place of beginning. Strictly speaking, in the word itself there is no necessary implication of regularity, but the word is often used loosely to denote rhythm or periodicity.

Rhythm, coming from a Greek word meaning measured time, implies a beat, or a tendency toward perfect regularity or periodicity. It

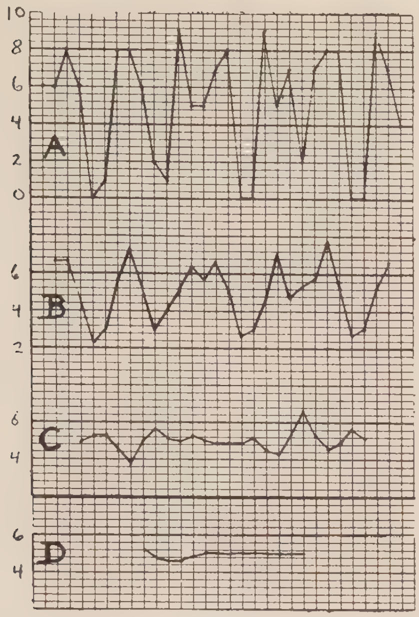


Fig. 1

A. Random Numbers
B. Their 3-Year Moving Average
C. Their 7-Year Moving Average
D. Their 16-Year Moving Average

Note that the longer the moving average, the smoother the curve.

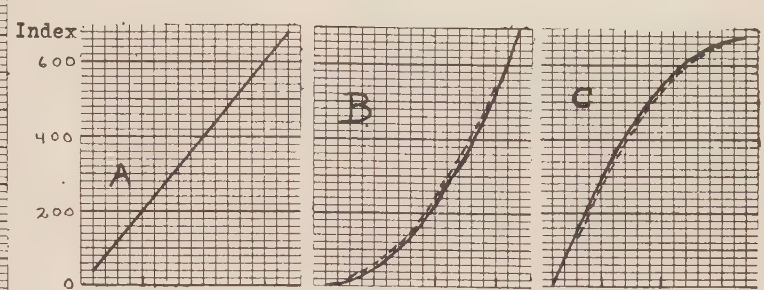


Fig. 2

A. Trend that increases by a constant amount together with its 5-year moving average. (The moving average does not show because it coincides with the trend.)
B. Trend that increases by increasing amounts and, broken line, its 5-year moving average. Note that the moving average lies above the trend.
C. Trend that increases by decreasing amounts and, broken line, its 5-year moving average. Note that the moving average lies below the trend.

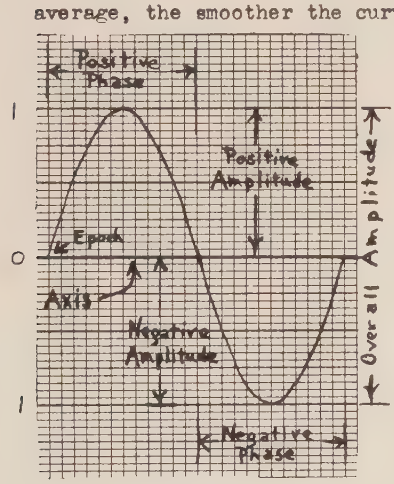


Fig. 3

Sine Wave

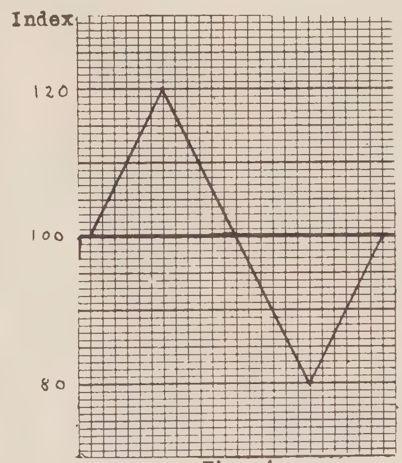


Fig. 4

Rectilinear Wave

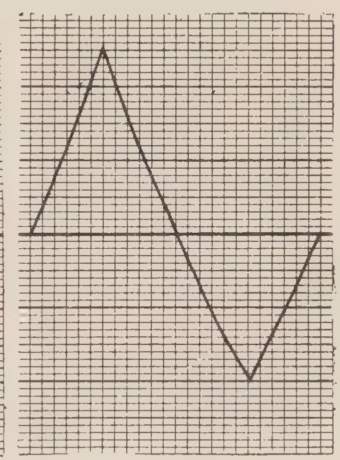


Fig. 5

"Compound Interest" Wave

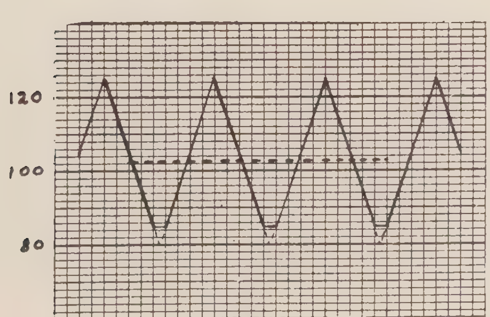


Fig. 6

9-Year Rectilinear Wave
and, broken line,
Its 9-Year Moving Average

Note that a 9-year wave is completely eliminated by a 9-year moving average.

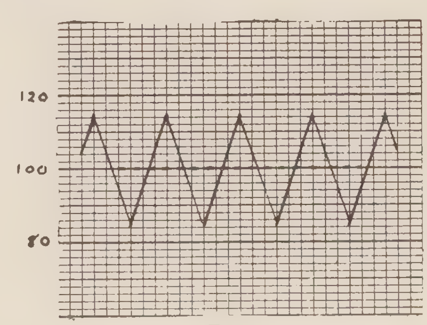


Fig. 7

6-Year Rectilinear Wave
and, broken line,
Its 6-Year Moving Average

Note that a 6-year wave is completely eliminated by a 6-year moving average.

is what we really meant on most of the occasions when we use the word cycle.

Cycle analysis, as we are using the term in this bulletin, should really be called rhythm analysis, as we are concerned with **rhythmic cycles**—cycles that recur with a beat.

Periodicity, in the strict sense, is the quality of being **regularly** recurrent. It is a quality not often found in nature. The ideal cycles that we shall presently construct for purposes of demonstration, however, are true periodicities.

A **wave** is one single cycle or undulation. Waves have **frequency**, **amplitude**, **period**, and, at least when they represent harmonic curves, **phase**.

Frequency is the number of complete vibrations to and fro—i.e. waves—per second. It is a term not used by cycle analysts when dealing with cycles that are over a second in length.

Amplitude is the range on one side or the other from the axis around which the wave oscillates. **Positive amplitude** is the distance above the axis, **negative amplitude** is the distance below the axis, **overall amplitude** is the sum of the positive and negative amplitudes. Amplitude may be expressed in absolute units or as a percentage of the axis or trend.

Period is the interval of time required for a periodic motion to complete a cycle and begin to repeat itself. It is the length of the wave from crest to crest or trough to trough or from some other point on the curve taken as the **epoch**. (The **epoch** is the point on the curve chosen as the beginning of the wave. In physics and astronomy it is usually taken as the point where the curve crosses the axis on its upward motion, but it may be any other point as well.)

Phase, in a simple harmonic curve, is the point or stage in the period to which the oscillation has advanced considered in relation to a standard position or assumed instant of starting. It is measured along the axis, usually in degrees. By extension of meaning, **positive phase** is therefore the part of the wave above the axis or trend, and **negative phase** is the part of the wave below the axis or trend. When the crests (or troughs) of two or more different series of waves come at the same time, the waves are said to be **in phase** with each other. When the crests of one series of waves coincides with the troughs of another series, the series are spoken of as **in reverse phase**.

A **simple harmonic curve** referred to once or twice above, is the curve you would get by tracing the motion of a pendulum upon a piece of smoked paper that was moving at uniform speed at right angles to the direction in which the pendulum was swaying back and forth. It is perfectly simple, regular, and symmetrical and in mathematical study, is usually referred to as a sine curve. A single oscillation is called a **sine wave**. This curve, and many of these definitions are illustrated in Fig. 3 on page 307.

A **rectilinear or saw-tooth wave**, on the other hand, is a wave the sides of which are straight lines; in other words, zigzag. See Fig. 4 on page 307. In electrical engineering a rectilinear wave usually refers to a square wave of this shape ⊓⊔⊓⊔, but the term has a more general application also.

Wave Shapes Usually Found

Because sine waves are so simple in shape, so easy to combine with each other and so satisfactory to handle mathematically, and because they are the shape taken by sound waves and many other kinds of waves with which the physicist deals, it is assumed by many students of cycles in climatology, biology, economics, and other fields that the waves with which they deal **ought** to be sine shape too.

Unfortunately things are not always what they "ought" to be. It has been my experience that waves in economic and biologic time series seem **never** to be sine shape (but this does not mean that the next wave I study might not be of this shape).

It is hard to be sure of the exact shape of a wave. There are almost always variations of length and of amplitude, as we go from one wave to the next. Also there are usually several rhythms present concurrently, and they mix each other up. Finally, there are random factors that enter into the picture which sometimes cannot be removed easily without distorting the wave shape. Therefore I cannot say I am sure of the **exact** mathematical average shape of the waves in any rhythm I have ever studied.

However, if I were forced to express my best guess, I would say that the waves we find in weather, biology, medicine, economics, hydrology, geology, etc., are likely, on the average, to be approximately rectilinear, that is saw-tooth or zigzag. It is not an

accident that the "ideal" waves that I have diagrammed in many of the charts that have been published are of this shape.

More exactly, I would put it that the **logarithms** of the data seem, on the average, to conform to a zigzag shape. The result of this fact, of course, is that the average wave shape is saw-tooth when the raw data are plotted on semi-logarithmic paper. This is another way of saying that the sides of the average wave seem to follow the shape of the compound interest curve. That is, the **percentage** rise from the trough to the axis is the same as the **percentage** rise from the axis to the crest

For example, if the trough is at 50 and the axis is at 100, the crest would be at 200 (not 150); one hundred is twice 50, and two hundred is twice one hundred. A wave that follows this law is illustrated in Fig. 5 on page 307.

(The characteristics just described offers another reason why it is usually so highly desirable, in subjecting a series of figures to a rhythm analysis, to convert the raw figures into logarithms before starting work, and to work with them throughout the course of the analysis.)

May I hasten to say that these beliefs are entirely the result of observations as to how the waves in general actually do behave, and are in no sense the result of theories as to how the waves "ought" to behave. I do not yet know enough to talk "oughts."

A second characteristic of the average waves of the rhythms I have studied is that with most of them the upward movements and the downward movements seem to be symmetrical. That is, the lows tend to fall midway between the highs, and vice versa. This characteristic is so generally true that I have come to suspect as possibly spurious any average wave which, without a reason, fails to conform to this pattern.

On the other hand, I have come across undoubted rhythms where the average waves were very definitely neither symmetrical nor of simple zigzag or compound interest form. It is not safe to try to generalize too rigidly.

In discussing the effect of moving averages upon periodic waves, I have chosen for illustration a perfectly symmetrical rectilinear or zigzag wave, because for small amplitude waves this is a close approximation of the typical form and is in fact seemingly the exact form when the data are converted to logarithms.

The Effect of Moving Averages upon Periodic Waves

1. Simple Waves

a. When the Lenght of the Moving Average is the Same as the Length of the Wave

Suppose yuo have a time series that evidences a perfectly regular 9-year cycle that repeats itself time after time as in Fig. 6 on page 307. The figures for the annual value of such a time series are given below.

Let us compute the 9-year moving average of this series of figures as in Col. C of Table 10.

It is obvious from reference to Col. C that the wave has disappeared and that the moving average is merely a straight line. This straight line has been plotted as a broken line in Fig. 6.

A moment's reflection will explain the reason for this behavior. As the length of the moving average is the same as the length of the wave, the value of the item that is added is always the same as the value of the item that is dropped and, in consequence, the moving average remains unchanged.

It is possible to generalize the above observation and to say that when the moving average has the same length as any perfectly regular wave, its effect is to eliminate the wave completely.

TABLE 10.
A 9-YEAR MOVING AVERAGE OF A 9-YEAR WAVE IN CONTROLLED DATA

	A	B	C
YEAR	CONTROLLED DATA EVIDENCING A 9-YEAR WAVE	9-YEAR MOVING TOT. OF DATA	9-YEAR MO. AV. OF THE DATA (COL. B ÷ 9; OR TIMES 1/9; OR TIMES .111111, THE RECIPROCAL OF 9
1ST	105	-	-
2ND	115	-	-
3RD	125	-	-
4TH	115	-	-
5TH	105	925	102.8
6TH	95	925	102.8
7TH	85	925	102.8
8TH	85	925	102.8
9TH	95	925	102.8
10TH	105	925	102.8
11TH	115	925	102.8
12TH	125	925	102.8
13TH	115	925	102.8
14TH	105	925	102.8
15TH	95	-	-
16TH	85	-	-
17TH	85	-	-
18TH	95	-	-

To make the procedure doubly plain and to pave the way for a discussion of a method of separating compound cycles, let us work another example. Table 11, next following, gives a series of figures evidencing a perfectly regular 6-year wave. The data are given, together with their centered 6-year moving average.

Here again you get a complete elimination of the wave. The 6-year moving average of the 6-year wave is merely a straight line. It is charted in Fig. 7 by means of a broken line superimposed upon the 6-year wave with which we started. (See p. 307.)

It should also be obvious that if you had added the 6-year wave to a trend line that increased by constant amounts, the 6-year moving average of the combined wave and trend line would have reproduced the trend free and clear of the wave. (If the trend had increased by increasing amounts, the moving average would have lain above it; if by decreasing amounts the moving average would have lain below it; all as illustrated in an earlier section.)

TABLE 11.
A 6-YEAR MOVING AVERAGE OF A 6-YEAR WAVE

YEAR	A DATA EVIDENCING A 6-YEAR WAVE	B 6-YEAR MOVING TOT. OF THE DATA POSTED TO THE 3RD POSITION	C 2-YEAR MOVING TOT. OF COL. B POSTED TO THE 2ND POSITION	D 6-YEAR MOVING AV. OF THE DATA CENTERED (COL. C÷12)
1ST	105	-	-	-
2ND	115	-	-	-
3RD	105	600	-	-
4TH	95	600	1200	100
5TH	85	600	1200	100
6TH	95	600	1200	100
7TH	105	600	1200	100
8TH	115	600	1200	100
9TH	105	600	1.200	100
10TH	95	-	-	-
11TH	85	-	-	-
12TH	95	-	-	-

Waves of Odd and Peculiar Shape

You may wonder if we would get the same result—a straight line—if the wave had some other shape. As long as the repetition is perfectly regular, the shape of the wave makes no difference whatever. This fact is illustrated in the table that follows:

TABLE 12.
A 5-YEAR MOVING AVERAGE OF AN IRREGULAR 5-YEAR WAVE

YEAR	A DATA EVIDENCING AN IRREGULAR SHAPED 5-YEAR REPETITIVE PATTERN	B 5-YEAR MOV. AVERAGE OF THE DATA
1ST	100	-
2ND	125	-
3RD	85	100
4TH	105	100
5TH	85	100
6TH	100	100
7TH	125	100
8TH	85	100
9TH	105	-
10TH	85	-

The reason we get a straight line is because the value we add is always the same as the value we drop.

b. When the Length of the Moving Average is An Integral Multiple of the Length of the Wave

A moving average that is two or three (or any other integral multiple) times the length of the wave will also completely eliminate any regular wave.

Thus, if we have a perfectly regular 9-year wave, an 18-year moving average will completely eliminate it, and so will a 27-year moving average, or a 36-year moving average.

If we have a perfectly regular 6-year wave, a 12-year, 18-year, 24-year, or 30-year moving average would give the same result.

You should also note that an 18-year moving average would completely eliminate **both** the 9-year and 6-year waves, because 18 is a multiple of both 9 years and 6 years.

c. When the Moving Average is of a Length That is Different From the Length of the Wave, or from some Integral Multiple of It.

You may wonder what a 3-year moving average of a 9-year wave might look like, or a 5-year moving average, or a 7-year moving average, or an 11-year moving average, or a 13-year moving average.

At this point the shape of the wave begins to make a difference. Let us therefore consider first the effect upon a rectilinear (saw tooth) wave. Such a wave is given in Table 13 on the page following, together with moving averages of various lengths. The various values are plotted in Fig. 8 on page 313.

You will note that as the moving averages get longer they become flatter until, when

the length of the moving average equals the length of the wave, the moving average becomes a straight line. When the moving average is longer than the length of the wave, the wave reappears in inverse (upside down) phase. That is, for the 11-year and 13-year moving averages, the 9-year wave reappears with troughs where there were crests in the original data, and with crests in the moving average where we originally had troughs.

The reason for this is very easy to see. The 13-year moving average, for example, centering on a trough, groups together two highs and one low and is therefore obviously **above** the average of one 9-year wave, at time of trough. As we progress in time to a position centering on a crest, the 13-year moving average includes two lows and one high and is therefore obviously **below** the average of one 9-year wave at time of crest.

TABLE 13.
VARIOUS MOVING AVERAGES OF A 9-YEAR WAVE

YEAR	A DATA EVIDENCING A REGULAR RECTILINEAR 9-YEAR WAVE	B 3-YEAR MOVING AVERAGE OF DATA	C 5-YEAR MOVING AVERAGE OF DATA	D 7-YEAR MOVING AVERAGE OF DATA	E 9-YEAR MOVING AVERAGE OF DATA	F 11-YEAR MOVING AVERAGE OF DATA	G 13-YEAR MOVING AVERAGE OF DATA
1ST	105		-	-	-	-	-
2ND	115	115.0	-	-	-	-	-
3RD	125	118.3	113	-	-	:	-
4TH	115	115.0	111	106.4	-	-	-
5TH	105	105.0	105	103.6	102.8	-	-
6TH	95	95.0	97	100.7	102.8	104.1	-
7TH	85	88.3	93	97.8	102.8	105.9	106.5
8TH	85	88.3	93	97.8	102.8	105.9	106.5
9TH	95	95.0	97	100.7	102.8	104.1	105.0
10TH	105	105.0	105	103.6	102.8	102.3	102.7
11TH	115	115.0	111	106.4	102.8	100.5	100.4
12TH	125	118.3	113	107.8	102.8	99.5	99.6
13TH	115	115.0	111	106.4	102.8	100.5	100.4
14TH	105	105.0	105	103.6	102.8	102.3	102.7
15TH	95	95.0	97	100.7	102.8	104.1	105.0
16TH	85	88.3	93	97.8	102.8	105.9	106.5
17TH	85	88.3	93	97.8	102.8	105.9	106.5
18TH	95	95.0	97	100.7	102.8	104.1	105.0
19TH	105	105.0	105	103.6	102.8	102.3	102.7
20TH	115	115.0	111	106.4	102.8	100.5	100.4
21ST	125	118.3	113	107.8	102.8	99.5	99.6
22ND	115	115.0	111	106.4	102.8	100.5	100.4
23RD	105	105.0	105	103.6	102.8	102.3	102.7
24TH	95	95.0	97	100.7	102.8	104.1	105.0
25TH	85	88.3	93	97.8	102.8	105.9	-
26TH	85	88.3	93	97.8	102.8	-	-
27TH	95	95.0	105	100.7	102.8	-	-
28TH	105	105.0	111	-	-	-	-
29TH	115	115.0	-	-	-	-	-
30TH	125	-	-	-	-	-	-

(HIGHS OF EACH CYCLE UNDERLINED)

d. Generalization for Rectilinear Waves

Fig. 9 was worked out by Benjamin Foote and James A. Mitchell of the Hartford Electric Light Company to generalize these facts for rectilinear waves. The chart was drawn for you by Mr. Mitchell. It gives you the percentage of the original amplitude remaining in the moving average for all simple arithmetic moving averages up to four times the length of the wave. (Fig. 9 will be found later in this bulletin on page 314.) This chart is a most useful one for all cycle analysts. I use mine constantly. Let us work out an example or two.

Two Examples

Suppose we have taken a 22-year moving average of a series of figures that contains a 17-year rectilinear (zigzag) wave. How much of the 17-year wave would remain in the 22-year moving average? Twenty-two is approximately 129.4% of 17. Find 129 on the horizontal scale at the bottom of Fig. 9. Construct a

perpendicular at this point. This perpendicular will cut the curved line at about minus 16 (read from the scale at position 50. The scale at the extreme left is for values on the horizontal scale from 0 to 50). There will therefore be minus 16% of the 17-year wave remaining in the moving average; that is, the wave in the moving averages will be in reverse phase or upside down from the wave in the original data.

Suppose we had taken a 38-year moving average of the same series of figures. How much of the original 17-year wave would be present in this 38-year moving average? Thirty-eight is 223.5% of 17. Therefore, we find 224 on our horizontal scale, construct a perpendicular. This perpendicular intersects the curve at plus 8 (read from the scale at position 50). Therefore, we know that 8% of the original amplitude of the 17-year rectilinear wave is still present in the 38-year moving average of these figures. If the amplitude of the moving average should prove to be 4, let us say, we could easily calculate that in the original figures it was 50 because 4 is 8% of 50.

Use of Tables

You may prefer to use a table instead of the chart. If so, you can refer to Table A below.

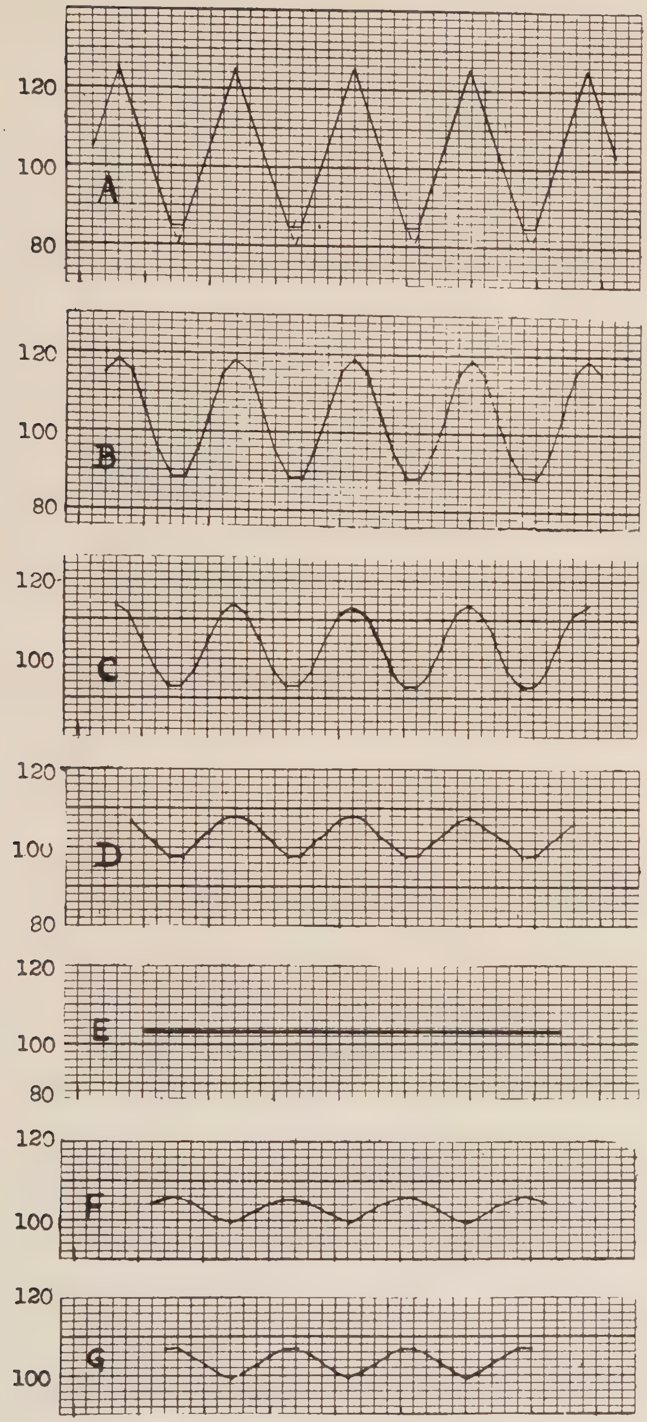
Let us work an example: Suppose we have a 23-year moving average of a regular zigzag shaped 54-year rhythm. How much of the rhythm remains in the moving average? Twenty three divided by 54 is 42.6%. Look up 42.6% in the first column in Table A—the column headed "The length of the moving average expressed as a percentage of the length of the wave." We find no value for 42.6 but we do find values for 40 and for 45. The percentage of the original amplitude remaining in the moving average for 40 is 60%, for 45 is 55%. By interpolation it is easy to compute that the correct percentage for 42.6% is 57.4%, the required answer.

TABLE A

PERCENTAGE OF AMPLITUDE OF ORIGINAL WAVE REMAINING IN A MOVING AVERAGE, WHEN THE WAVE IS REGULAR, SYMMETRICAL, AND RECTILINEAR OR SAW-TOOTH IN SHAPE, FOR VARIOUS LENGTHS OF MOVING AVERAGES UP TO FOUR TIMES THE LENGTH OF THE WAVE.

The Length of the Moving Average Expressed as a Percentage of the Length of the Wave	Percentage of Original Amplitude Remaining in the Moving Average	A Cont'd.	B Cont'd.	A Cont'd.	B Cont'd.	A Cont'd.	B Cont'd.
0	100.	100	.0	200	.0	300	.0
5	95.	105	-4.5	205	2.3	305	-1.6
10	90.	110	-8.2	210	4.3	310	-2.9
15	85.	115	-11.1	215	5.9	315	-4.0
20	80.	120	-13.3	220	7.3	320	-5.0
25	75.	125	-15.0	225	8.3	325	-5.8
30	70.	130	-16.2	230	9.1	330	-6.4
35	65.	135	-16.8	235	9.7	335	-6.8
40	60.	140	-17.1	240	10.0	340	-7.1
45	55.	145	-17.1	245	10.1	345	-7.2
50	50.	150	-16.7	250	10.0	350	-7.1
55	45.	155	-16.0	255	9.7	355	-7.0
60	40.	160	-15.0	260	9.2	360	-6.7
65	35.	165	-13.8	265	8.6	365	-6.2
70	30.	170	-12.4	270	7.8	370	-5.7
75	25.	175	-10.7	275	6.8	375	-5.0
80	20.	180	-8.9	280	5.7	380	-4.2
85	15.	185	-6.9	285	4.5	385	-3.1
90	10.	190	-4.7	290	3.1	390	-2.3
95	5.	195	-2.4	295	1.6	395	-1.2
100	0.	200	0.0	300	0.0	400	0.0

Fig. 8



A. A series of 9-year rectilinear waves

B. Their 3-year moving average

C. Their 5-year moving average

D. Their 7-year moving average

E. Their 9-year moving average

F. Their 11-year moving average

G. Their 13-year moving average

Note how, as the moving average gets longer the waves get flatter until, when the length of the moving average is the same as the length of the wave, they disappear. As the moving average gets longer still, the waves reappear in reverse phase (upside down).

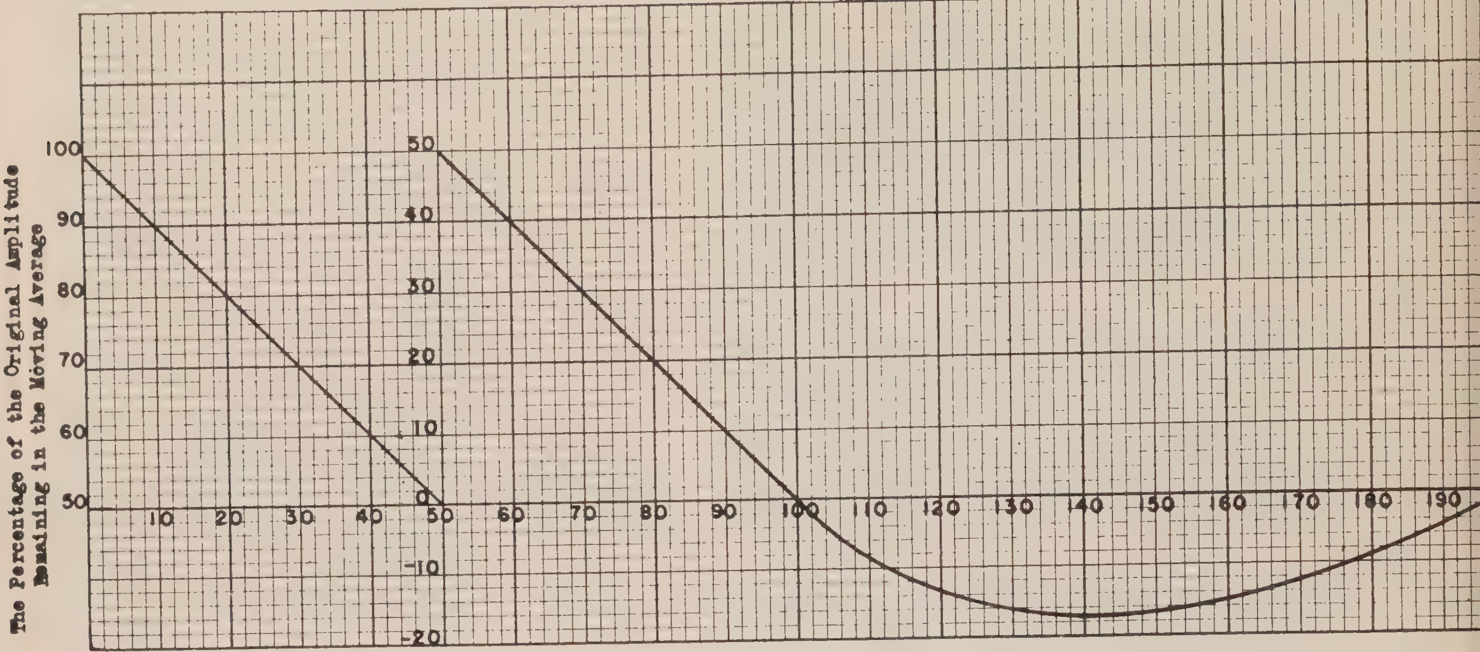


FIG. 9. FOR RECTILINEAR (SAW-TOOTH) WAVES — The Length of the

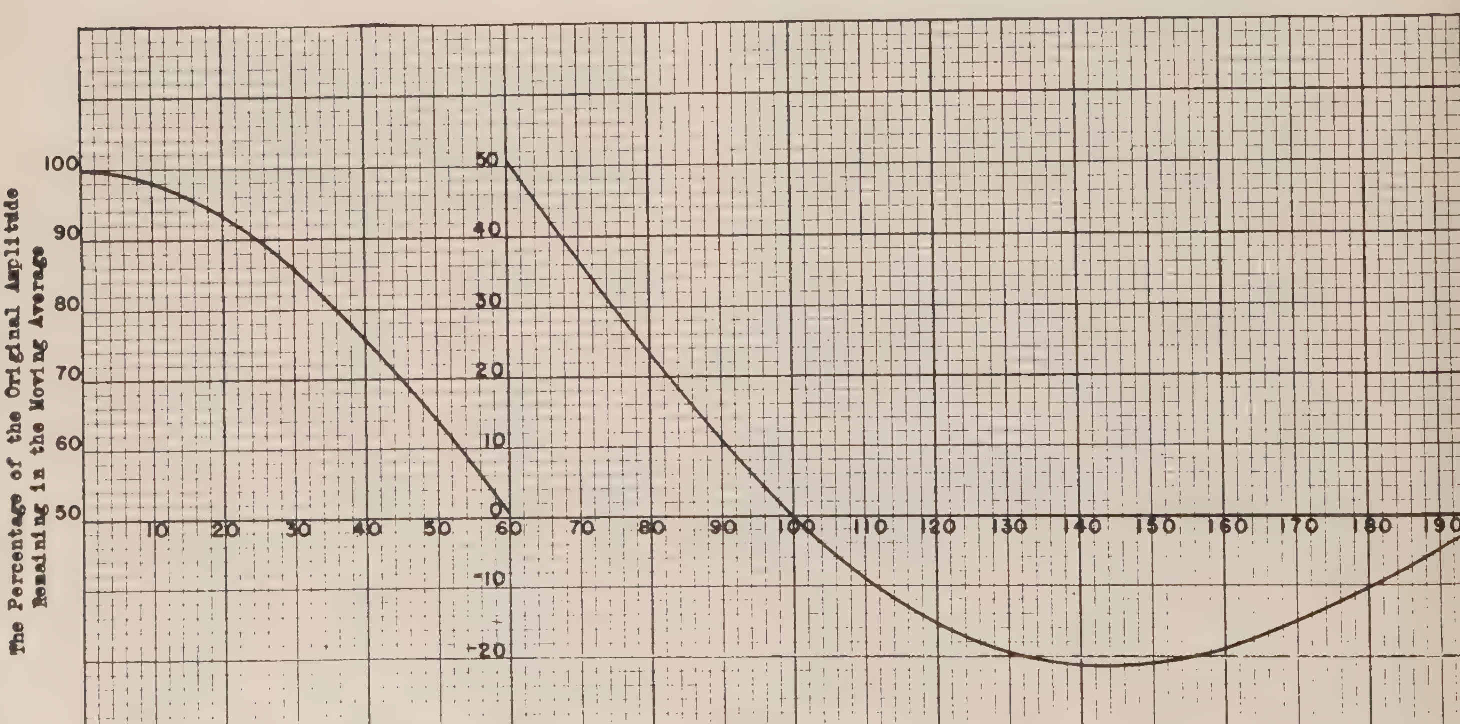
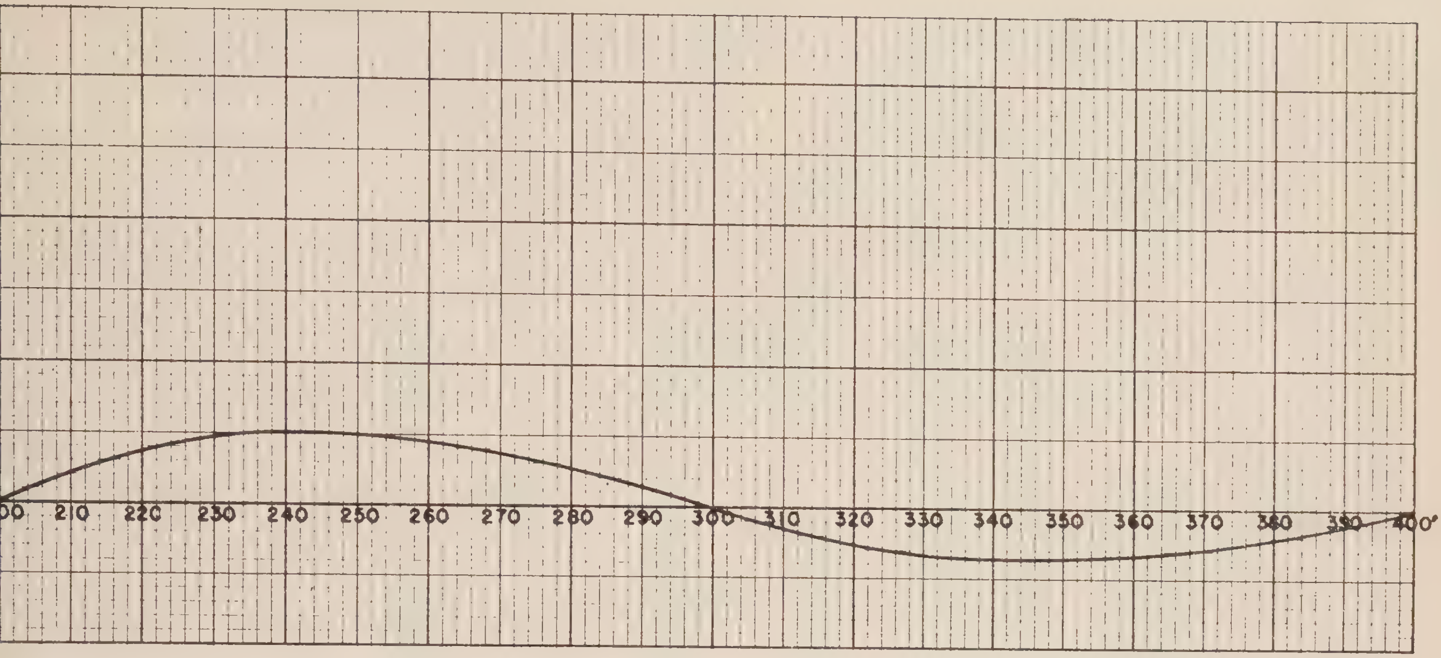
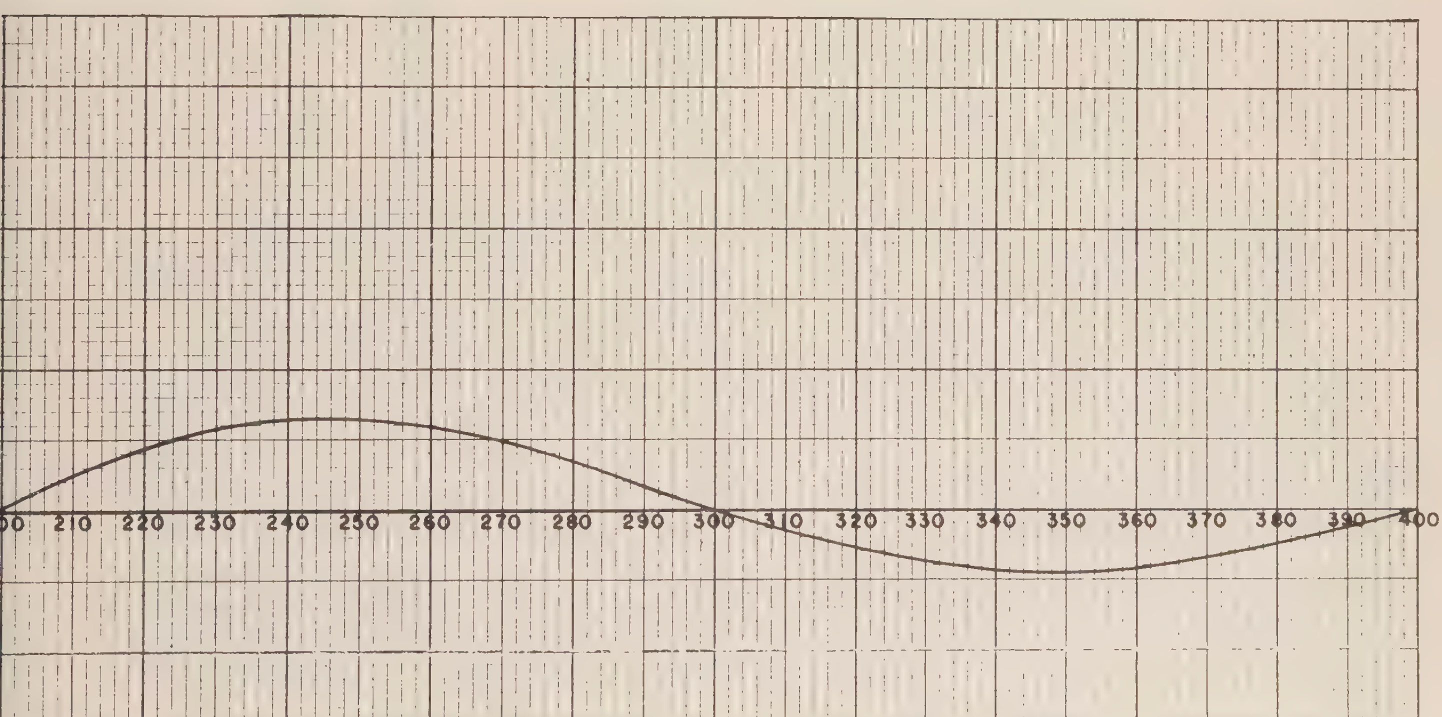


FIG. 10. FOR SINE WAVES — The Length of the Movi



Moving Average Expressed as a Percentage of the Length of the Rhythm



Average Expressed as a Percentage of the Length of the Rhythm

e. Generalization for Sine Waves

At this point you may ask, does this chart hold true for sine waves and for other waves that are not rectilinear or zigzag shape? No, it does not. Each shape of wave requires a separate diagram. For sine waves Mr. Foote and Mr. Mitchell constructed Fig. 10, inserted previously in this bulletin on page 314, to show the percentage of the original amplitude remaining in the moving average of a sine wave for all given lengths of moving average up to four times the length of the wave.

If you prefer to use a table, refer to Table B below.

TABLE B

PERCENTAGE OF AMPLITUDE OF ORIGINAL WAVE REMAINING IN A MOVING AVERAGE,
WHEN THE WAVE IS REGULAR, SYMMETRICAL, AND SINE SHAPED
FOR VARIOUS LENGTHS OF MOVING AVERAGES UP TO FOUR TIMES THE LENGTH OF THE WAVE

A The Length of the Moving Average Expressed as a Percentage of the Length of the Wave	B Percentage of Original Amplitude Remaining in the Moving Average	A Cont'd.	B Cont'd.	A Cont'd.	B Cont'd.	A Cont'd.	B Cont'd.
0	.0	100	.0	200	.0	300	.0
5	99.4	105	-4.7	205	2.4	305	-1.6
10	98.4	110	-8.9	210	4.7	310	-3.2
15	96.4	115	-12.6	215	6.7	315	-4.6
20	93.6	120	-15.6	220	8.5	320	-5.8
25	90.1	125	-18.0	225	10.0	325	-6.9
30	85.9	130	-19.8	230	11.2	330	-7.8
35	81.0	135	-21.0	235	12.1	335	-8.5
40	75.7	140	-21.6	240	12.6	340	-8.9
45	69.9	145	-21.7	245	12.8	345	-9.1
50	63.7	150	-21.2	250	12.7	350	-9.1
55	57.2	155	-20.3	255	12.3	355	-8.9
60	50.5	160	-18.9	260	11.6	360	-8.4
65	43.6	165	-17.2	265	10.7	365	-7.8
70	36.8	170	-15.1	270	9.5	370	-7.0
75	30.0	175	-12.9	275	8.2	375	-6.0
80	23.4	180	-10.4	280	6.7	380	-4.9
85	17.0	185	-7.8	285	5.1	385	-3.7
90	10.9	190	-5.2	290	3.4	390	-2.5
95	5.2	195	-2.5	295	1.7	395	-1.2
100	0.	200	0.	300	0.	400	0.

2. Compound Waves

Moving Averages of Time Series Influenced by Two or More Concurrent Cycles

Let us now add together the two series of figures containing the 6-year wave and the 9-year wave respectively that we dealt with above and which were charted in Figs. 6 and 7. This addition is performed in the table on the following page and the result is plotted in Fig. 11 on page 319.

This sum is the kind of a pattern one might expect in the total sales of a company that were equally divided between two products, the sales of one of which fluctuated with a 9-year rhythm, and a second product, the sales of which fluctuated with a 6-year rhythm.

TABLE 14.
6-YEAR, 9-YEAR, AND 18-YEAR MOVING AVERAGES OF A COMPOUND WAVE

	A	B	C	D	E	F
YEAR	DATA EVIDENCING THE 9-YEAR CYCLE	DATA EVIDENCING THE 6-YEAR CYCLE	SUM OF COL. A AND COL. B	6-YEAR MOVING AVERAGE OF COL. C, CENTERED	9-YEAR MOVING AVERAGE OF COL. C	18-YEAR MOVING AVERAGE OF COL. C, CENTERED
1ST	105	105	210	-	-	-
2ND	115	115	230	-	-	-
3RD	125	105	230	-	-	-
4TH	115	95	210	208.3	-	-
5TH	105	85	190	204.2	205.6	-
6TH	95	95	190	199.2	204.4	-
7TH	85	105	190	195.8	201.1	-
8TH	85	115	200	195.8	200.0	-
9TH	95	105	200	199.2	201.1	-
10TH	105	95	200	204.2	204.4	202.8
11TH	115	85	200	208.3	205.6	202.8
12TH	125	95	220	210.0	204.4	202.8
13TH	115	105	220	208.3	201.1	202.8
14TH	105	115	220	204.2	200.0	202.8
15TH	95	105	200	199.2	201.1	202.8
16TH	85	95	180	195.8	204.4	202.8
17TH	85	85	170	195.8	205.6	202.8
18TH	95	95	190	199.2	204.4	202.8
19TH	105	105	210	204.2	201.1	202.8
20TH	115	115	230	208.3	200.0	202.8
21ST	125	105	230	210.0	201.1	202.8
22ND	115	95	210	208.3	204.4	202.8
23RD	105	85	190	204.2	205.6	202.8
24TH	95	95	190	199.2	204.4	202.8
25TH	85	105	190	195.8	201.1	202.8
26TH	85	115	200	195.8	200.0	202.8
27TH	95	105	200	199.2	201.1	202.8
28TH	105	95	200	204.2	204.4	-
29TH	115	85	200	208.3	205.6	-
30TH	125	95	220	210.0	204.4	-
31ST	115	105	220	208.3	201.1	-
32ND	105	115	220	204.2	201.1	-
33RD	95	105	200	199.2	-	-
34TH	85	95	180	-	-	-
35TH	85	85	170	-	-	-
36TH	95	95	190	-	-	-

(HIGHS OF EACH CYCLE UNDERLINED)

Let us now take these figures that evidence this composite wave and compute first a 9-year moving average, second, a 6-year moving average and third, an 18-year moving average as in the table on the preceding page. The various moving averages are plotted in Fig. 11 on page 319. The 9-year moving average has the effect of completely eliminating the 9-year component of the series and shows the 6-year wave in reverse phase, that is to say, with tops where bottoms used to be and vice versa, but with reduced amplitude, all as we would expect from the foregoing discussion.

The 6-year moving average has the effect of completely eliminating the 6-year wave and leaving the 9-year wave in proper phase position (that is, with tops of the moving average where there were tops in the data, and bottoms in the moving average where there were bottoms in the data), but with greatly reduced amplitude.

The 18-year moving average of course eliminates both 6-year and 9-year waves, and would also have eliminated any 4½-year wave (¼ of 18 years), any 3.6-year wave (1/5 of 18 years), any 3-year wave (1/6 of 18 years) and so on if there had been such in the original data. By the same token, it would have revealed any wave longer than 18 years, or shorter waves that were not integral fractions of the length of 18 years, or both, if these had also been present in the data.

Discussion

It should be clear from the foregoing demonstrations that every time you take a moving average of a series of figures, you are performing an operation that has an effect upon the amplitude, and sometimes reverses the phase, of all the regularly recurring waves that may be present in the original figures. It is this fact that prompts the Celticism that one is really not able to start a rhythmic analysis until after one has finished it.

In other words, until one knows the length of all the waves that are present in a series, one is not fully in a position to choose the lengths of the moving averages to use to emphasize some and subordinate others.

Comparison of the Raw Data With with the Moving Average

In the section which began on page 309, you had demonstrated for you the fact that when the length of the moving average is the same as the length of the wave, the effect is the complete elimination of the wave.

Where the original data consist of nothing but a wave (and a horizontal trend line) as in Tables 10 and 11, it is obvious that if we compare the original data with the moving average (which is a horizontal straight line) the result will merely reconstitute the wave in its entirety. This fact is illustrated in Curve EE of Fig. 12 on page 321 and in Col. EE of Table 15 on page 320.

When the moving average is of a length which differs from the length of the wave in the original data, or some multiple of it, some part of the original wave will remain in the moving average. This residue of the original wave remaining in the moving average will be either in phase with the original wave or in reverse phase (upside down). All of this was demonstrated in Table 13 and illustrated in Fig. 8.

When the moving average retains some of the wave **in phase** with the original wave, and when the original data are compared with such a moving average, it should be clear that the difference between the two will show the original wave with **reduced** amplitude. For example, if we have a 9-year wave with an amplitude of 10, and the moving average also contains a 9-year wave coming at the same time with an amplitude of 2, the series of figures evidencing the difference between the two waves will show an amplitude of 8.

When the moving average shows the wave **in reverse phase**, or upside down, and when the original data are compared with it, the difference between the two will show the original wave with **increased** amplitude. For example, if the wave just discussed with an amplitude of 10 were being compared with a moving average that was of such a length that it evidenced a 9-year wave with an amplitude of -1, the wave in the series of figures evidencing the difference would show an amplitude of 11. These facts are all illustrated for the moving averages given in Table 13, in Fig. 12 and in Table 15 on page 320.

It should be noted that the comparisons above have been made by subtraction for the sake of si plicity. In actual practice one ordinarily makes the comparison by division and determines the **percentages** that the original data are of their moving average.

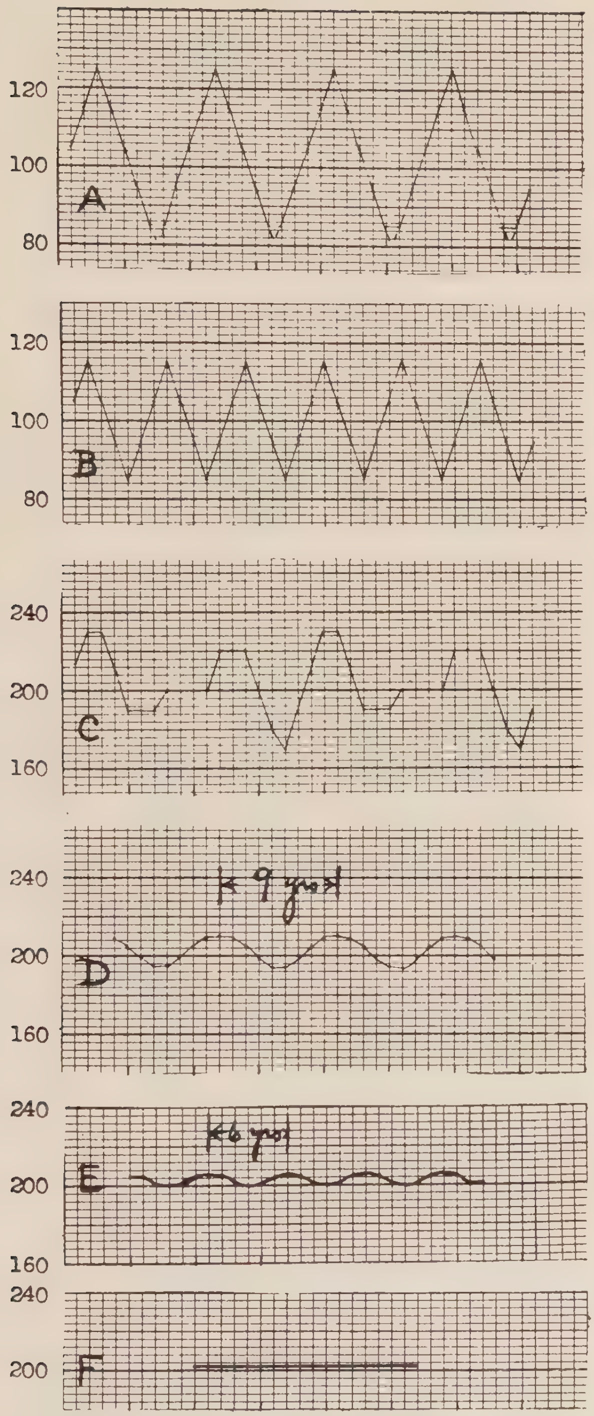
There are several reasons for making the comparisons on a percentage basis. One has already been mentioned—the fact that mostly the waves seem to be the same **percentage** above and below the axis. When one uses percentages on real waves therefore, one tends to get waves that are symmetrical with respect to the axis.

A second reason is that in actual practice, most waves with which one deals are superimposed upon trend lines. That is, the phenomenon with which we deal, let us say the abundance of lynx or the thickness of tree rings or the size of a business, has a long term increase or decrease over a period of time. Experience indicates that the waves that are associated with these various phenomena are usually of approximately constant **percentage** amplitude.

A third reason for making percentage comparisons, even if the trend should be horizontal, is that if there should be other waves it will usually be found that the various waves combine by multiplication and not by addition. They therefore must be unscrambled by division instead of by subtraction.

A fourth reason for making comparison on a percentage basis is that in making projections for most series, one must talk of the waves in terms of percentages. " If the wave continues, the sales of the company at such

Fig. 11



A. A series of 9-year waves

B. A series of 6-year waves

C. Their summation (A + B)

D. A 6-year moving average of the summation (reveals a 9-year wave in phase with the original 9-year wave

E. A 9-year moving average of the summation (reveals a 6-year wave in <u>reverse</u> phase from the original 6-year wave (upside down)).

F. An 18-year moving average of the summation (completely eliminates both waves)

and such a time will be 10% above its then trend line." Where the trend line will be at the time must be computed separately.

The only exceptions to the above rule that occurs to me at the moment are (a) the case where the waves in the original data are expressed in plus and minus values, and (b) the case where some of the values of the raw data are zero. In these instances, comparisons between the moving average and the data should usually be made by subtraction.

TABLE 15.

COMPARISON OF ORIGINAL CONTROLLED DATA WITH VARIOUS MOVING AVERAGES

	A	B	BB	C	CC	D	DD	E	EE	F	FF	G	GG
Year	Data Evidencing a Regular Saw-tooth 9-Year Wave	3-Year Moving Average of the Data	Data Divided by Their 3-Year Moving Average (%)	5-Year Moving Average of the Data	Data Divided by Their 5-Year Moving Average (%)	7-Year Moving Average of the Data	Data Divided by Their 7-Year Moving Average (%)	9-Year Moving Average of the Data	Data Divided by Their 9-Year Moving Average (%)	11-Year Moving Average of the Data	Data Divided by Their 11-Year Moving Average (%)	13-Year Moving Average of the Data	Data Divided by Their 13-Year Moving Average (%)
1st	105	-	-	-	-	-	-	-	-	-	-	-	-
2nd	115	115.0	100.0	-	-	-	-	-	-	-	-	-	-
3rd	125	118.3	105.7	113.0	110.6	-	-	-	-	-	-	-	-
4th	115	115.0	100.0	111.0	103.6	106.4	108.1	-	-	-	-	-	-
5th	105	105.0	100.0	105.0	100.0	103.6	101.4	102.8	102.1	-	-	-	-
6th	95	95.0	100.0	97.0	97.9	100.7	94.3	102.8	92.4	104.1	91.3	-	-
7th	85	88.3	96.3	93.0	91.4	97.8	86.9	102.8	82.7	105.9	80.3	106.5	79.8
8th	85	88.3	96.3	93.0	91.4	97.8	86.9	102.8	82.7	105.9	80.3	106.5	79.8
9th	95	95.0	100.0	97.0	97.9	100.7	94.3	102.8	92.4	104.1	91.3	105.0	90.5
10th	105	105.0	100.0	105.0	100.0	103.6	101.4	102.8	102.1	102.3	102.6	102.7	102.2
11th	115	115.0	100.0	111.0	103.6	106.4	108.1	102.8	111.9	100.5	114.4	100.4	114.5
12th	125	118.3	105.7	113.0	110.6	107.8	116.0	102.8	121.6	99.5	125.6	99.6	125.5
13th	115	115.0	100.0	111.0	103.6	106.4	108.1	102.8	111.9	100.5	114.4	100.4	114.5
14th	105	105.0	100.0	105.0	100.0	103.6	101.4	102.8	102.1	102.3	102.6	102.7	102.2
15th	95	95.0	100.0	97.0	97.9	100.7	94.3	102.8	92.4	104.1	91.3	105.0	90.5
16th	85	88.3	96.3	93.0	91.4	97.8	86.9	102.8	82.7	105.9	80.3	106.5	79.8
17th	85	88.3	96.3	93.0	91.4	97.8	86.9	102.8	82.7	105.9	80.3	106.5	79.8
18th	95	95.0	100.0	97.0	97.9	100.7	94.3	102.8	92.4	104.1	91.3	105.0	90.5
19th	105	105.0	100.0	105.0	100.0	103.6	101.4	102.8	102.1	102.3	102.6	102.7	102.2
20th	115	115.0	100.0	111.0	103.6	106.4	108.1	102.8	111.9	100.5	114.4	100.4	114.5
21st	125	118.3	105.7	113.0	110.6	107.8	116.0	102.8	121.6	99.5	125.6	99.6	125.5
22nd	115	115.0	100.0	111.0	103.6	106.4	108.1	102.8	111.9	100.5	114.4	100.4	114.5
23rd	105	105.0	100.0	105.0	100.0	103.6	101.4	102.8	102.1	102.3	102.6	102.7	102.2
24th	95	95.0	100.0	97.0	97.9	100.7	94.3	102.8	92.4	104.1	91.3	105.0	90.5
25th	85	88.3	96.3	93.0	91.4	97.8	86.9	102.8	82.7	105.9	80.3	106.5	79.8
26th	85	88.3	96.3	93.0	91.4	97.8	86.9	102.8	82.7	105.9	80.3	106.5	79.8
27th	95	95.0	100.0	97.0	97.9	100.7	94.3	102.8	92.4	104.1	91.3	-	-
28th	105	105.0	100.0	105.0	100.0	103.6	101.4	102.8	102.1	-	-	-	-
29th	115	115.0	100.0	111.0	103.6	106.4	108.1	-	-	-	-	-	-
30th	125	118.3	105.7	113.0	110.6	-	-	-	-	-	-	-	-
31st	115	115.0	100.0	-	-	-	-	-	-	-	-	-	-
32nd	105	-	-	-	-	-	-	-	-	-	-	-	-

(ALL CRESTS UNDERLINED)

It will be noted both in the chart and in the table that when we compare the original data with their various moving averages that the rhythm that was present in the original data continues present in the comparison. That is, if the wave in the original data is 9 years long, and we run a 7-year moving average through the series (Col. D above) and compare the original data with the 7-year moving average (Col. DD), we get the 9-year rhythm with

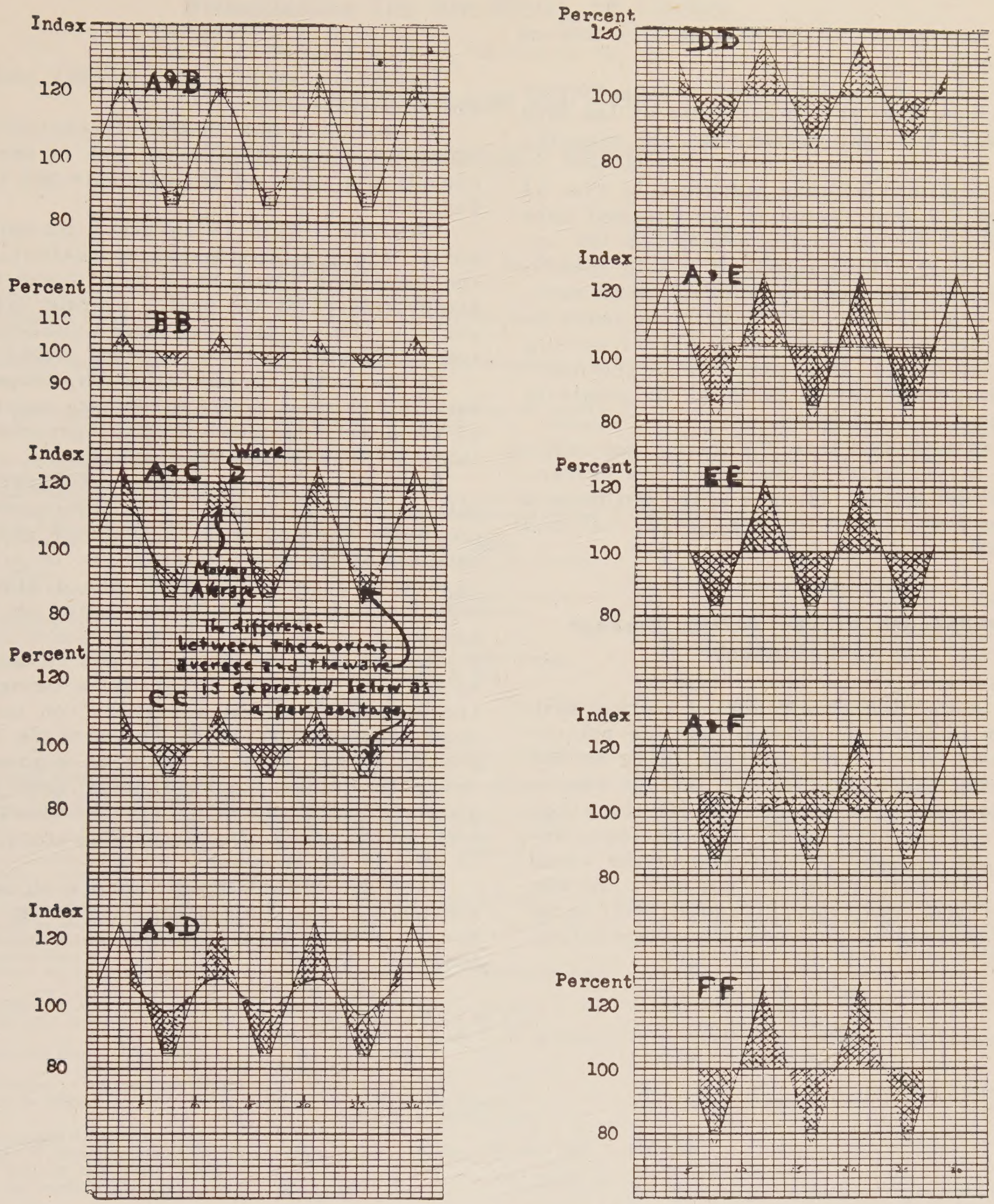


Fig. 12

A & B. 9-Year Rectilinear Wave Together With Its 3-Year Moving Average

BB. Data Divided by Their 3-Year Moving Average

A & C. 9-Year Rectilinear Wave Together With Its 5-Year Moving Average

CC. Data Divided by Their 5-Year Moving Average

A & D. 9-Year Rectilinear Wave Together With Its 7-Year Moving Average

DD. Data Divided by Their 7-Year Moving Average

A & E. 9-Year Rectilinear Wave Together With Its 9-Year Moving Average

EE. Data Divided by Their 9-Year Moving Average

A & F. 9-Year Rectilinear Wave Together With Its 11-Year Moving Average

FF. Data Divided by Their 11-Year Moving Average

Note: Note that as the number of terms in the moving average increase and the moving average gets flatter, the original wave reappears more and more in the percentages. When the moving average equals the length of the wave, the wave reappears fully. As the number of items in the moving average increase further, the amplitude of the original wave is magnified.

Note also that the percentages that the data are of their moving averages always evidence waves of the length of the wave in the original data and not the length of the moving average.

which we started, albeit with greatly reduced amplitude.

In other words, the 7-year moving average of the data does **not** in any sense of the word **introduce** a 7-year rhythm into the comparisons.

The converse of this statement is that if we have a 9-year rhythm in the original data and take a 9-year moving average of the series, compare the original data with this 9-year moving average and find a 9-year wave, the 9-year wave we find can in no sense be construed as a **result** of a 9-year moving average, either. This seems to be the hardest thing about moving averages for people to realize.

It is suggested that you prove these statements to yourself by computing the percentages that actual figures which evidence a rhythm are of moving averages of various lengths.

Comparison of One Moving Average with Another

The comparison of one moving average with another is merely an extension of the principles that have been explained fully in the foregoing pages. One moving average can be used to eliminate one or more of the minor waves and minimize random fluctuations, another can be used to approximate the trend line. The comparison of the two, if the lengths have been properly chosen, will often reveal or emphasize waves of intermediate length.

Summary

The moving average is a useful tool for cycle analysts.

By smoothing out random fluctuations and shorter cycles, it aids the eye to see more clearly the waves of intermediate and longer length.

When the length is suitably chosen, the moving average provides an approximation of the underlying growth trend with, however, the disadvantage that the moving average will lie above or below the true trend, unless the trend is increasing by a constant amount.

By the proper choice of length, the moving average can often effect a complete separation of two interacting wave systems present concurrently in the same time series.

By means of the technique of first computing the moving average of a series and then computing the percentages that the original data are of the moving average, it is possible to obtain a curve in which the distorting effect of trend and of longer cycles are minimized.

The use of moving averages can be compared to the use of color filters on a camera. By the proper choice of a filter, you can reveal characteristics of the article being photographed, such as grain in a piece of wood, that might be completely lost in an ordinary photograph and might be overlooked even by the naked eye. The moving average can be used in the same way.

Used with intelligence, and with a full knowledge of its limitations, the moving average is a very valuable tool for the cycle analyst.

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